Mathematical Appendix

I. Proof of proposition 1:

We will note $\Delta = \{(\gamma,\delta) \,|\, \delta + \gamma \leq 1 \,;\, \delta \geq 0 \,\&\, \gamma \geq 0 \}$ and n_i the number of agents of type *i*.

To compute to mean payoffs of each type we need first to compute the probability for an agent to be connected.

<u>Lemma 1</u> : Let *i* be an agent taken at random in a sub-population *P* of agents which are looking for some particular type(s), in proportion λ in the population, to do relations; the probability that *i* has already done some relations is $P_{connect}(\lambda) = 1 - \frac{1-\theta}{1-(1-\lambda)\theta}$

Proof :

Let *i* be an agent of age *t* in *P*. The probability it hasn't met any agent to do relations during its life is $(1-\lambda)^t$. Consequently, the probability that agent *i* of age *t* has a non empty address book is $1-(1-\lambda)^t$. (We neglect here the occurrences when your only relation dies before you meet an other one, which will be a good approximation as soon as *e* is not too small).

The distribution of ages in the population has the generatrice function $f(z) = (1-\theta)\sum_t \theta^t z^t$. Then the probability for an agent to be connected has the generatrice function $\phi(z) = \left(1 - \frac{1-\theta}{1-(1-\lambda)\theta}\right) z + 1 - \frac{1-\theta}{1-(1-\lambda)\theta}$ λ. We deduce the mean connectedness with agents of the considered types(s): $P_{connect}(\lambda) = 1 - \frac{1-\theta}{1-(1-\lambda)\theta}$ and its variance

$$Var_{connect}(\lambda) = \left(1 - \frac{1-\theta}{1-(1-\lambda)\theta}\right)\frac{1-\theta}{1-(1-\lambda)\theta} . \bullet$$

In the area $p/r < \delta$, the domain concerned by emergence of cooperation, only altruists cooperate first move. Consequently, they are the only one to be able to do relations with altruist *and* reciprocators. Their probability to be connected is then $P_{connect}(\delta+\gamma)$. Selfish agents and reciprocators, on the contrary, can only do relations with altruists. Their probability to be connected is then $P_{connect}(\gamma)$. We can now compute the payoffs.

Payoffs terms are composed of two independent terms: the expected payoffs as first mover and the expected payoffs as second mover. For each of them we have to treat separately the cases where the agent is connected, the cases where it is not and exploration moves. We then get:

$$\pi_{selfish}=P_{connect}(\gamma)[(1\text{-}e)+e.p(1\text{-}\gamma)+e.\gamma]+\ldots \qquad \# 1$$

$$(1\text{-}P_{connect}(\gamma))[p.(1\text{-}\gamma)+\gamma]+\ldots \qquad \# 2$$

$$e.[P_{connect}(\gamma).(1\text{-}\gamma).p+P_{connect}(\gamma+\delta).\gamma]+\ldots \qquad \# 3$$

$$(1\text{-}P_{connect}(\gamma)).(1\text{-}\gamma).p+(1\text{-}P_{connect}(\gamma+\delta)).\gamma \qquad \# 4$$

With :

1 : *i* is connected, 1rst move interactions

2 : i is not connected, 1rst move interactions at random

3 : *2nd* move interactions with connected agents

4 : 2nd move interactions with not connected agents

After simplifications we get:

$$\pi_{selfish}=(1-e)\left(\frac{2p(1-\gamma+\theta(\gamma-1))+\gamma}{1+\theta(\gamma-1)}+\frac{(1-\theta)\gamma}{1-(1-(\delta+\gamma))\theta}\right)+2e(p(1-\gamma)+\gamma)\ (\pi_s) \qquad \text{(Eq. 1)}$$

In the same way :

$$\pi_r=P_{connect}(\gamma)((1\text{-}e).r+e.(\gamma+p(1\text{-}\gamma)))+\ldots \qquad \# 1$$

$$(1\text{-}P_{connect}(\gamma))(p.(1\text{-}\gamma)+\gamma)+\ldots \qquad \# 2$$

$$e.(P_{connect}(\gamma).(1\text{-}\gamma).p+P_{connect}(\gamma+\delta).[(1\text{-}e).r.\gamma/(\gamma+\delta)+e.\gamma.r])+\ldots \qquad \# 3$$

$$(1\text{-}P_{connect}(\gamma)).(1\text{-}\gamma).p+(1\text{-}P_{connect}(\gamma+\delta)).\gamma.r \qquad \# 4$$

After simplification we get :

$$\pi_{reciprocators} = (1-e)\left(\frac{\theta\gamma(r+2p-1)+\gamma+2p(1-\gamma-\theta)}{1+\theta(\gamma-1)} + \frac{r\gamma}{1-(1-(\delta+\gamma))\theta}\right) + e(2p+\gamma(1-2p+r) \quad (\pi_r) \qquad \text{(Eq. 2)}$$

and

$$\pi_a = P_{connect}(\gamma+\delta)[(1-e).r+e.r.(\gamma+\delta)]+\ldots \qquad \# 1$$

$$(1-P_{connect}(\gamma+\delta)).(\delta+\gamma).r+\ldots \qquad \# 2$$

$$P_{connect}(\gamma).(1-e).r.\delta/\gamma+P_{connect}(\gamma+\delta).[(1-e).r.\gamma/(\gamma+\delta)+e.\gamma.r]+\ldots \qquad \# 3$$

$$(1-P_{connect}(\gamma+\delta)).\gamma.r \qquad \# 4$$

After simplifications we get:

$$\pi_{alruists} = (1-e)\left(\frac{r\theta\delta}{1+\theta(\gamma-1)} + \frac{r(2\gamma+\delta)}{1-(1-(\delta+\gamma))\theta}\right) + er(\delta+2\gamma) \quad (\pi_a) \qquad \text{(Eq. 3)}$$

II. Theoretical activity

To compute the theoretical mean activity, we follow the same strategy as for the expected payoffs with the difference that we have to take in consideration interactions for which payoffs are *0.*

Mean activity for selfish agents:

Mean activity of agents can be split into two independents terms: the mean activity as first mover and the mean activity as second mover. For each of them we have to treat separately the cases where the agent is connected, the cases where it is not and exploration moves. We then get:

$$\pi_{selfish} = 1 \qquad \# 1 + \# 2$$

$$e.[P_{connect}(\gamma).(1-\gamma)+P_{connect}(\gamma+\delta).\gamma]+\ldots \qquad \# 3$$

$$(1-P_{connect}(\gamma)).(1-\gamma)+(1-P_{connect}(\gamma+\delta)).\gamma \qquad \# 4$$

With :

1 : *i* is connected, 1rst move interactions

2 : i is not connected, 1rst move interactions at random

3 : 2^{nd} move interactions with connected agents

 # 4 : 2^{nd} move interactions with not connected agents

In the same way for reciprocators:

$\pi_r = 1+$ #1 + # 2

$e.P_{connect}(\gamma).(1-\gamma)+ P_{connect}(\gamma+\delta).[(1-e).\gamma/(\gamma+\delta)+e.\gamma])+\ldots$ # 3

$(1-P_{connect}(\gamma)).(1-\gamma)+(1- P_{connect}(\gamma+\delta)).\gamma$ # 4

and for altruists :

In the same way for reciprocators :

$\pi_a = 1+$ #1 + # 2

$P_{connect}(\gamma).[e.(1-\gamma)+(1-e).\ \delta/\gamma]+P_{connect}(\gamma+\delta).[e.\gamma+(1-e).\ \gamma/(\gamma+\delta)]+\ldots$ # 3

$(1-P_{connect}(\gamma)).(1-\gamma)+(1- P_{connect}(\gamma+\delta)).\gamma$ # 4

Complements

For the domain $\frac{p}{1-p} < \delta$, things are a little more complex. Since this area is not concerned by emergence of cooperation, we will only give some guidelines, with highlight on differences with the case treated above. As selfish agent cooperates first move, it may have a successful interaction with an altruist or a reciprocator. But the expected payoff associated with an altruist relation is higher than the one associated with a reciprocator. In fact, if a selfish agent *i* knows an altruist *j*, *i* will defect with *j* every times *i* will have the choice to do so. Therefore; the expected payoffs are $(1-e)\frac{1}{(1-\theta^2)}$ On the other hand, expected payoffs associated with a reciprocator relation are $(1-e)\frac{1-p}{(1-\theta^2)}$. It could be in its interest for a selfish agent to check if a given new relation is an altruist by defecting at the second interaction. Let us compute the expected payoffs of these two strategies:

1. S1: If it doesn't check, the expected payoffs are $1-p+(1-e)\theta^2\frac{1-p}{(1-\theta^2)}$

2. S2: If it does check, the expected payoffs are :

$$\frac{\delta}{\delta+\gamma}\left(p+(1-e)\theta^2\frac{1-p}{(1-\theta^2)}\right)+\frac{\gamma}{\delta+\gamma}\left(1+(1-e)\frac{\theta^2}{(1-\theta^2)}\right)$$

The difference is then :

$$S2-S1=\frac{\delta}{\delta+\gamma}(2p-1)+\frac{\gamma}{\delta+\gamma}\left(p+(1-e)\frac{\theta_2}{(1-\theta_2)}p\right)$$

$$S2-S1\geq 0\Leftrightarrow\delta\frac{(2p-1)}{p\left(1+(1-e)\frac{\theta_2}{(1-\theta_2)}\right)}\leq\gamma \qquad \text{(Eq. 3)}$$

The coefficient of δ in Eq. 3 is very low and consequently, $S2-S1\geq 0$ is true for almost the whole area $\frac{p}{1-p}<\delta$. We will expose here only the case *S2-S1>0.*

Then we have to compute for the special case of selfish agents, probability of finding the different types of address books: 1) empty, 2) with reciprocator(s) relations but no altruist relation, 3) with altruist(s) relations. After that, with the same type of calculus as in 2.1, we find that in simplified form :

$$\pi_{egoists}=(1-e)\left(\frac{2p(1-\gamma+\theta(\gamma-1))+\gamma}{1+\theta(\gamma-1)}+\frac{(1-\theta)\gamma}{1-(1-(\delta+\gamma))\theta}\right)+2e(p(1-\gamma)+\gamma)$$

$$\pi_{reciprocators}=(1-e)\left(\frac{\theta p\gamma+2p(1-\gamma-\theta))}{1+\theta(\gamma-1)}+\frac{(1-p)\gamma}{1-(1-(\delta+\gamma))\theta}\right)+e(2p+2\gamma-3p\gamma)$$

$$\pi_{alruists}=(1-e)\left(\frac{(1-p)\theta\delta}{1+\theta(\gamma-1)}+\frac{(1-p)(2\gamma+\delta)}{1-(1-(\delta+\gamma))\theta}\right)+e(\delta(1-p)+2\gamma-2p\gamma)$$

Computational Appendix

We give here the algorithm used for the simulations.

Parameters and notation

The parameters are:

1. $e \in [0,1]$: Exploration parameter
2. $p \in [0, \frac{1}{2}]$: Game parameter
3. $\theta \in [0, \frac{1}{2}]$: Parameter defining the mean lifetime expectancy.

The population at time *t* is noted *P(t).*

Initial conditions :

N agents with empty address books from the three types: altruist, reciprocator and selfish, in proportion δ, γ and $(1-\gamma-\delta)$.

At each period of time t :

Set all payoffs to 0

Interaction (parallel updating):

1. For each agent of *P(t)*, an individual binomial random variable defines if it will be allowed with a probability *1-e* to chose its partner. If its address book is empty, the agent has to interact with an unknown partner as first mover.
2. In case of interaction as first mover with an unknown partner, the partner is chosen at random and table 1 gives the action played.
3. In case of interaction as first mover in address book:
    - Altruists and reciprocators choose a relation at random and play *C*.
    - Selfish look for an altruist and defect. If they know only reciprocators, they choose one at random and cooperate. In they don't know altruists but know agents with

ambiguous types (a (C,C) result in the preceding interaction as first mover), they choose one of them at random and defect to test whether it is an altruist. (This situation only happens in the area $\frac{p}{1-p} < \delta$).

4. Each agent replies as second mover according to its type (cf. table 1).
5. Each first mover adds the name of its second mover partner in its address book if the partner has cooperated. Selfish have special categories: a *(D,C)* results - *altruist,* a *(C,C)* result – *reciprocator or altruist*, a *(D,D)* result after interaction with *reciprocator or altruist* – *reciprocator.*

Selection :

1. After interactions took place, the total payoffs of each agent are computed by adding the payoffs of all two persons games they were engaged in this period.
2. Each agent dies with a probability $(1-\theta)$.
3. Died agents are replaced with new agents with empty address books. Died agents are suppressed from the address books of their friends. This lead to populations *P(t+1).*
4. Types of new agents are determined by a replicator dynamics indexed on the total payoffs of agents from *P(t)*: for each new born agent, an agent of *P(t)* is selected with a probability proportional to its total payoffs. The new agent inherits the type of this particular agent.